R & D NOTES

Isothermal Steady Spinning of an Oldroyd Fluid B

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INTRODUCTION

Mathematical modeling of fiber spinning from polymeric melts is receiving a great deal of attention. Kase and Matsuo (1965) initiated the mathematical simulation of melt spinning for Newtonian fluids. Matovich and Pearson (1969) studied the steady state spinning of Newtonian and Coleman-Noll second-order fluids. Pearson and Shah (1974) extended their studies to power-law fluids. Kase (1974) investigated the spinning process in which melt viscosity was dependent on fiber radial and longitudinal directions. Zeichner (1973) obtained an approximation solution for steady state spinning of Maxwell fluids.

Perhaps the most important contribution in mathematical modeling has come from the work of Denn et al. (1975), who took into account the large stresses that are developed in the elongation flows of polymeric melts. A modified Oldroyd fluid, the Sun-Denn model, was used to describe the melt behavior. Denn and coworkers (with Fisher, 1976, 1977; Marucci, 1977; and Gagon, 1981) extended this work by employing a variety of rheological models. Petrie (1978, 1979) also investigated the modeling of the melt spinning process. White and Ide (1978) studied the isothermal spinning of a White-Metzner fluid, but assumed that the relaxation time followed the Bogue-White equation. Recently, Larson (1983) analyzed fiber spinning with a Doi-Edwards constitutive equation.

In this short note, we attempt to extend Denn's works to simulate the melt spinning of an Oldroyd fluid B model.

GOVERNING EQUATIONS

Assuming that the fluid is incompressible and isotropic, and that inertia, gravity, and surface tension are negligible, the equations for conservation of mass and energy during the isothermal steady-state melt spinning are

$$AV = Q \tag{1}$$

$$A(\tau_{11} - \tau_{22}) = F \tag{2}$$

Here, A is the cross-sectional area of the filament, V is the axial velocity, Q is the volume flow rate of the extruded melt, and τ_{11} and τ_{22} are the normal stresses in the axial and the radial directions, respectively. F is the force required to draw the filaments. Dieswell is not considered in this analysis.

For an Oldroyd fluid B, its rheological behavior may be defined as (Bird et al. 1977)

$$\tau + \lambda_1 \frac{\delta \tau}{\delta t} = \eta \left[\dot{\gamma} + \lambda_2 \frac{\delta \dot{\gamma}}{\delta t}\right] \tag{3}$$

where τ is the extrastress tensor and $\dot{\gamma}$ is the rate-of-strain tensor. λ_1 and λ_2 are the relaxation and retardation times of the fluid, respectively (Oldroyd, 1956). $\delta/\delta t$ is Oldroyd's convected derivative for contravariant tensors defined elsewhere (Denn et al., 1975; Bird et al., 1977).

Neglecting certain shearing terms and employing the continuity equation, we obtain the following equations for τ_{11} and τ_{22}:

$$\tau_{11} + \lambda_1 \left(V \frac{d\tau_{11}}{dx} - 2\tau_{11}\frac{dV}{dx}\right) = 2\eta \left\{\frac{dV}{dx} + \lambda_2 \left[v \frac{d^2V}{dx^2} - 2\left(\frac{dV}{dx}\right)^2\right]\right\} \tag{4}$$

and

$$\tau_{22} + \lambda_1 \left(V \frac{d\tau_{22}}{dx} + \tau_{22}\frac{dV}{dx}\right) = -\eta \left\{\frac{dV}{dx} + \lambda_2 \left[\frac{d^2V}{dx^2} + \left(\frac{dV}{dx}\right)^2\right]\right\} \tag{5}$$

Where the x coordinate starts at a position defined by Denn et al. (1975). The following dimensionless variables and parameters are defined as

$$U = V/V_0,\ \xi = x/L,\ T = \tau_{11}Q/FV_0,\ P = \tau_{22}Q/FW_0$$

$$\alpha_1 = \frac{\lambda_1 V_0}{L},\ \alpha_2 = \frac{\lambda_2 V_0}{L},\ \epsilon = \frac{\eta Q}{FL}$$

Substituting these dimensionless terms into Eqs. 2, 4, and 5, and rearranging the resulting equations yield

$$T = \frac{U}{3\alpha\dfrac{dU}{d\xi}} + \frac{2U}{3} - \frac{\epsilon}{\alpha_1} - \frac{\epsilon\alpha_2 U}{\alpha_1}\frac{\dfrac{d^2U}{d\xi^2}}{\dfrac{dU}{d\xi}} + \frac{\epsilon\alpha_2}{\alpha_1}\frac{dU}{d\xi} \tag{6}$$

Equation 4 may be rewritten as follows, with the aid of Eq. 6:

$$U + (\alpha_1 U - 3\epsilon)\frac{dU}{d\xi} + (3\epsilon\alpha_2 - 2\alpha_1^2 U)\left(\frac{dU}{d\xi}\right)^2 + 6\epsilon\alpha_1\alpha_2\left(\frac{dU}{d\xi}\right)^3 - 3\epsilon\alpha_2 U\frac{d^2U}{d\xi^2} - \alpha_1 U^2 \frac{\dfrac{d^2U}{d\xi^2}}{\dfrac{dU}{d\xi}} + 3\epsilon\alpha_1\alpha_2 U^2 \frac{\left(\dfrac{d^2U}{d\xi^2}\right)^2}{\dfrac{dU}{d\xi}} - 3\epsilon\alpha_1\alpha_2 U^2 \frac{d^3U}{d\xi^3} = 0 \tag{7}$$

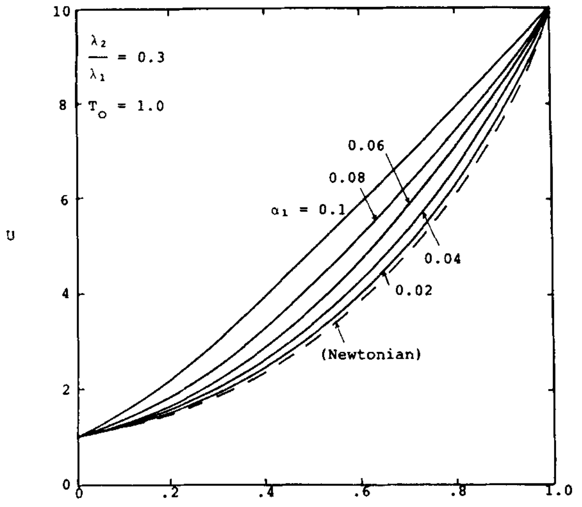

Figure 1. Velocity profile as a function of dimensionless relaxation time.

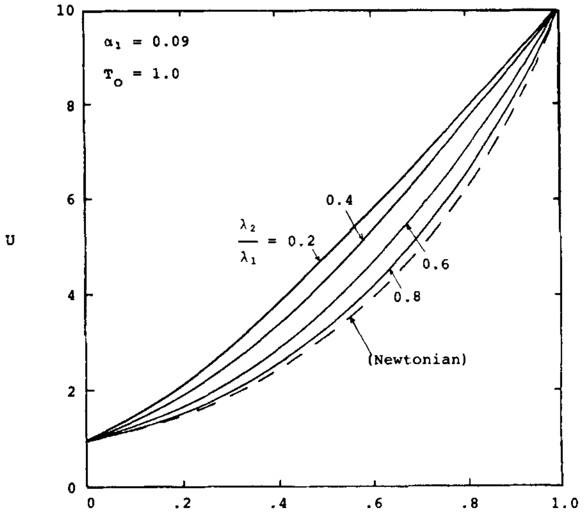

Figure 2. Velocity profile as a function of the ratio of λ_2/λ_1.

This equation is a special case of the previous studies reported by Petrie (1978, 1979) and can be solved using a forward integration technique if the three boundary conditions at $\xi = 0$ are defined. One of them is

$$U = 1 \text{ at } \xi = 0 \tag{8}$$

Others are very difficult to define. Based on the previous literature (Denn et al., 1975), and the magnitude of each item in both Eqs. 6 and 7, the following assumptions for boundary conditions are introduced:

$$T = T_0 \text{ at } \xi = 0$$

$$\frac{d^2U}{d\xi^2} = \left(\frac{dU}{d\xi}\right)^2 \text{ at } \xi = 0 \tag{10}$$

Equation 9 indicates that the stress at $\xi = 0$ is fixed, and Eq. 10 comes from the fact that the magnitude of $d^2U/d\xi^2$ and $dU/d\xi$ are comparable. They are equal in the case of Netwonian fluid. Substitution Eqs. 8 through 10 into Eq. 6 yields the value of $dU/d\xi$ at $\xi = 0$.

SOLUTION

In the case of Newtonian fluid, $\lambda_1 = \lambda_2$, the solution of Eq. 10 is

$$U = \exp(\xi/3\epsilon) \tag{11}$$

There is no analytical solution when λ_1 and λ_2 are different. Equation 10 can be easily solved using a fourth-order Runge-Kutta method. The value of ϵ is varied until the value of U at $\xi = 1$ matches the specified drawdown ratio D_R.

RESULTS AND DISCUSSION

The effect of the dimensionless relaxation parameter α_1 on the velocity profile is shown in Figure 1. Here, the Newtonian result was calculated from Eq. 11 with a known draw ratio. The relation between α_1 and the velocity profile is very similar to that obtained by Denn et al. (1975), Fisher and Denn (1977), and White and Ide (1978) for a variety of modified Maxwell fluid models. In addition, computer simulation results indicate that the velocity profile down the spinline is strongly dependent on the tension force term, ϵ, but weakly on the choice of T_o. This observation confirms that the last two terms in Eq. 6 during the calculation of $dU/d\xi$ at $\xi = 0$ can be neglected.

Figure 2 illustrates the effect of (λ_2/λ_1) on the spinline velocity. When (λ_2/λ_1) is zero, the melt viscoelastic behavior is that of a codeformational Maxwell fluid. Comparison of Figure 1 to Figure 2 clearly indicates that the retardation parameter α_2 has the tendency to counterbalance the relaxation parameter α_1 in melt viscoelastic behavior. In addition, because both figures predict the results in accord with the previous publications, we can conclude that Oldroyd fluid B model is capable of predicting the melt-spinning process with a proper choice of the relaxation and retardation time constants.

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NOTATION

D_R = draw ratio
F = force
L = filament length
P = dimensionless extrastress in x direction
Q = volumetric flow rate
T = dimensionless extrastress in z direction
T_o = initial value of T
U = dimensionless axial velocity component
V = axial velocity component
V_o = initial velocity
Z = axial direction

Greek Letters

α_1 = dimensionless relaxation parameter
α_2 = dimensionless retardation parameter
$\dot{\gamma}$ = rate of strain tensor
ϵ = dimensionless reciprocal force
λ_1 = relaxation time
λ_2 = retardation time
η = zero-shear viscosity
ξ = dimensionless axial position
τ = extrastress tensor
τ_{11} = normal stress (extra stress) in x direction
τ_{22} = normal stress (extra stress) in z direction

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